Contents

Preface

This book grew out of Harvard University's honors freshman mechanics course. It is essentially two books in one. Roughly half of each chapter follows the form of a normal textbook, consisting of text, along with exercises suitable for homework assignments. The other half takes the form of a "problem book," with all sorts of problems (and solutions) of varying degrees of difficulty. I've always thought that doing problems is the best way to learn, so if you've been searching for a supply to puzzle over, I think this will keep you busy for a while.

This book is somewhat of a quirky one, so let me say right at the start how I imagine it being used:

- As the primary text for honors freshman mechanics courses. My original motivation for writing it was the fact that there didn't exist a suitable book for Harvard's freshman course. So after nine years of using updated versions in the class, here is the finished product.
- As a supplementary text for standard freshman courses for physics majors. Although this book starts at the beginning of mechanics and is self contained, it doesn't spend as much time on the introductory material as other freshman books do. I therefore don't recommend using this as the only text for a standard freshman mechanics course. However, it will make an extremely useful supplement, both as a problem book for all students, and as a more advanced textbook for students who want to dive further into certain topics.
- As a supplementary text for upper-level mechanics courses, or as the primary text which is supplemented with another book for additional topics often covered in upper-level courses, such as Hamilton's equations, fluids, chaos, Fourier analysis, electricity & magnetism applications, etc. With all of the worked examples and in-depth discussions, you really can't go wrong in pairing up this book with another one.
- As a problem book for anyone who likes solving physics problems. This audience ranges from advanced high school students, who I think will have a ball with it, to undergraduate and graduate students who want some amusing problems to ponder, to professors who are looking for a new supply of problems to use in their classes, and finally to anyone with a desire to learn about physics by doing problems. If you want, you can consider this to be a problem book that also happens to have comprehensive introductions to each topic's set of problems. With about 250 problems (with included solutions) and 350 exercises (without included solutions), in addition to all the examples in the text, I think you'll get your money's worth! But just in case, I threw in 600 figures, 50 limericks, nine appearances of the golden ratio, and one cameo of $e^{-\pi}$.

The prerequisites for the book are solid high school foundations in mechanics (no electricity and magnetism required) and single-variable calculus. There are two minor exceptions

to this. First, a few sections rely on multivariable calculus, so I have given a review of this in Appendix B. The bulk of it comes in Section 5.3 (which involves the curl), but this section can easily be skipped on a first reading. Other than that, there are just some partial derivatives, dot products, and cross products (all of which are reviewed in Appendix B) sprinkled throughout the book. Second, a few sections (4.5, 9.2–9.3, and Appendices D and E) rely on matrices and other elementary topics from linear algebra. But a basic understanding of matrices should suffice here.

A brief outline of the book is as follows. Chapter 1 discusses various problem solving strategies. This material is extremely important, so if you read only one chapter in the book, make it this one. You should keep these strategies on the tip of your brain as you march through the rest of the book. Chapter 2 covers statics. Most of this will likely be familiar, but you'll find some fun problems. In Chapter 3, we learn about forces and how to apply $F = ma$. There's a bit of math here needed for solving some simple differential equations. Chapter 4 deals with oscillations and coupled oscillators. Again, there's a fair bit of math needed for solving linear differential equations, but there's no way to avoid it. Chapter 5 deals with conservation of energy and momentum. You've probably seen much of this before, but it has lots of neat problems.

In Chapter 6, we introduce the Lagrangian method, which will most likely be new to you. It looks rather formidable at first, but it's really not all that rough. There are difficult concepts at the heart of the subject, but the nice thing is that the technique is easy to apply. The situation here is analogous to taking a derivative in calculus; there are substantive concepts on which the theory rests, but the act of taking a derivative is fairly straightforward.

Chapter 7 deals with central forces and planetary motion. Chapter 8 covers the easier type of angular momentum situations, where the direction of the angular momentum vector is fixed. Chapter 9 covers the more difficult type, where the direction changes. Spinning tops and other perplexing objects fall into this category. Chapter 10 deals with accelerating reference frames and fictitious forces.

Chapters 11 through 14 cover relativity. Chapter 11 deals with relativistic kinematics – abstract particles flying through space and time. Chapter 12 covers relativistic dynamics – energy, momentum, force, etc. Chapter 13 introduces the important concept of "4-vectors." The material in this chapter could alternatively be put in the previous two, but for various reasons I thought it best to create a separate chapter for it. Chapter 14 covers a few topics from general relativity. It's impossible for one chapter to do this subject justice, of course, so we'll just look at some basic (but still very interesting) examples. Finally, the appendices cover various useful, but slightly tangential, topics.

Throughout the book, I have included many "Remarks." These are written in a slightly smaller font than the surrounding text. They begin with a small-capital "Remark" and end with a shamrock (♣). The purpose of these remarks is to say something that needs to be said, without disrupting the overall flow of the argument. In some sense these are "extra" thoughts, although they are invariably useful in understanding what is going on. They are usually more informal than the rest of the text, and I reserve the right to use them to occasionally babble about things that I find interesting, but that you may find tangential. For the most part, however, the remarks address issues that arise naturally in the course of the discussion. I often make use of "Remarks" at the ends of the solutions to problems, where the obvious thing to do is to check limiting cases (this topic is discussed in Chapter 1). However, in this case, the remarks are *not* "extra" thoughts, because checking limiting cases of your answer is something you should *always* do.

For your reading pleasure (I hope!), I have included limericks throughout the text. I suppose that these might be viewed as educational, but they certainly don't represent any deep insight I have into the teaching of physics. I have written them for the sole purpose of lightening things up. Some are funny. Some are stupid. But at least they're all physically

accurate (give or take).

As mentioned above, this book contains a huge number of problems. The ones with included solutions are called "Problems," and the ones without included solutions, which are intended to be used for homework assignments, are called "Exercises." There is no fundamental difference between these two types, except for the existence of written-up solutions. I have chosen to include the solutions to the problems for two reasons. First, students invariably want extra practice problems, with solutions, to work on. And second, I had a thoroughly enjoyable time writing them up. But a warning on these problems and exercises: Some are easy, but many are very difficult. I think you'll find them quite interesting, but don't get discouraged if you have trouble solving them. Some are designed to be brooded over for hours. Or days, or weeks, or months (as I can attest to!).

The problems (and exercises) are marked with a number of stars (actually asterisks). Harder problems earn more stars, on a scale from zero to four. Of course, you may disagree with my judgment of difficulty, but I think an arbitrary weighting scheme is better than none at all. As a rough idea of what I mean by the number of stars, one-star problems are solid problems that require some thought, and four-star problems are really, really, *really* hard. Try a few and you'll see what I mean. Even if you understand the material in the text backwards and forwards, the four-star (and many of the three-star) problems will still be extremely challenging. But that's how it should be. My goal was to create an unreachable upper bound on the number (and difficulty) of problems, because it would be an unfortunate circumstance if you were left twiddling your thumbs, having run out of problems to solve. I hope I have succeeded.

For the problems you choose to work on, be careful not to look at the solution too soon. There's nothing wrong with putting a problem aside for a while and coming back to it later. Indeed, this is probably the best way to learn things. If you head to the solution at the first sign of not being able to solve a problem, then you have wasted the problem.

REMARK: This gives me an opportunity for my first remark (and first limerick, too). One thing many people don't realize is that you need to know more than the correct way(s) to do a problem; you also need to be familiar with many *incorrect* ways of doing it. Otherwise, when you come upon a new problem, there may be a number of decent-looking approaches to take, and you won't be able to immediately weed out the poor ones. Struggling a bit with a problem invariably leads you down some wrong paths, and this is an essential part of learning. To understand something, you not only have to know what's right about the right things; you also have to know what's wrong about the wrong things. Learning takes a serious amount of effort, many wrong turns, and a lot of sweat. Alas, there are no shortcuts to understanding physics.

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Any book that takes ten years to write is bound to contain the (greatly appreciated) input of many people. I am particularly thankful for Howard Georgi's help over the years, with his numerous suggestions, ideas for many problems, and physics sanity checks. I would also like to thank Don Page for his entertaining and meticulous comments, and an eye for catching errors in earlier versions. Other friends and colleagues who have helped make this book what it is (and who have made it all the more fun to write) are John Bechhoefer, Wes Campbell, Michelle Cyrier, Alex Dahlen, Gary Feldman, Lukasz Fidkowski, Jason Gallicchio, Bertrand Halperin, Matt Headrick, Jenny Hoffman, Paul Horowitz, Alex Johnson, Yevgeny Kats, Ben Krefetz, Daniel Larson, Rakhi Mahbubani, Megha Padi, Dave Patterson, Konstantin Penanen, Courtney Peterson, Mala Radhakrishnan, Esteban Real,

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Finally, and perhaps most importantly, I would like to thank all the students (both at Harvard and elsewhere) who provided input during the past decade. The names here are literally too numerous to write down, so let me simply say a big thank you, and that I hope other students will enjoy what you helped create.

Happy problem solving!

David Morin
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