

Contents

This book is divided into two volumes. The Table of Contents includes both volumes.

Volume 1

Preface	ix
1 Operations, fractions	1
1.1 Products	2
1.2 Quotients	5
1.3 Order of operations	6
1.4 Negative numbers	8
1.5 Sums and differences	10
1.6 Operations vs. labels	13
1.7 Products involving negative numbers	16
1.8 The negative of a negative	18
1.9 Parentheses	21
1.10 Inequalities	23
1.11 Fractions	26
1.12 Multiplying and dividing fractions	30
1.12.1 General case	30
1.12.2 Special cases	39
1.12.3 Further topics	42
1.13 Adding fractions	48
1.14 Exercise solutions	54
2 Laws of algebra, exponents	66
2.1 Three laws	66
2.1.1 Commutative law	67
2.1.2 Associative law	70
2.1.3 Distributive law	75
2.2 Exponents	80
2.2.1 Definition	80
2.2.2 Rules	82
2.2.3 Observations	85

2.2.4	Other bases	88
2.2.5	Fractional powers	90
2.2.6	Negative powers	96
2.3	Prime factorization	98
2.3.1	Primes	98
2.3.2	Prime factorization	101
2.4	LCM, GCF	105
2.4.1	LCM	105
2.4.2	GCF	107
2.4.3	Fractions	109
2.5	Exercise solutions	112
3	Expansions, FOIL	129
3.1	Collecting like terms	129
3.2	FOIL	134
3.3	Three special cases	138
3.4	Trinomials, etc.	146
3.5	Cubes, etc.	150
3.6	Binomial expansion	155
3.7	Approximations	158
3.8	Exercise solutions	163
4	Factoring, applications	175
4.1	Do's and don'ts	175
4.2	Factoring	179
4.2.1	Single terms	179
4.2.2	Binomials	182
4.3	Roots, zeros	188
4.3.1	Obtaining roots from factors	188
4.3.2	Obtaining factors from roots	190
4.4	Polynomial long division	195
4.5	Many examples and exercises	199
4.6	Being general	205
4.7	Exercise solutions	209
5	Solving for x	224
5.1	Equal changes to equal things	225
5.2	Undoing the operations	237
5.3	More than one x	242
5.4	Inequalities	244
5.5	Word problems	248
5.5.1	Translating words into equations	248
5.5.2	Train problems	253
5.6	Systems of equations	257

5.6.1	Two equations, two unknowns	257
5.6.2	Age problems	261
5.6.3	Inconsistent, degenerate equations	263
5.6.4	Three equations, three unknowns	266
5.7	Quadratic equations	269
5.7.1	Solving by factoring	269
5.7.2	Systems of equations	272
5.7.3	Quadratic inequalities	274
5.8	Exercise solutions	277
6	Coordinate systems and lines	294
6.1	1 Dimension	294
6.2	2 Dimensions	296
6.3	3 Dimensions	302
6.4	Lines	307
6.4.1	Plotting lines	307
6.4.2	The slope m	310
6.5	Further line considerations	314
6.5.1	Plotting tools, Desmos	314
6.5.2	Perpendicular slope	316
6.5.3	Systems of equations	318
6.6	Different expressions for lines	320
6.7	Distance from a point to a line	323
6.7.1	Perpendicular-line method	323
6.7.2	Similar-triangle method 1	326
6.7.3	Similar-triangle method 2	330
6.8	Exercise solutions	333
A	Geometry tools	345
A.1	Angles	345
A.1.1	Degrees	345
A.1.2	Vertical angles	347
A.1.3	Parallel lines, alternate interior angles	349
A.2	Triangles	352
A.2.1	180° in a triangle	352
A.2.2	Types of triangles	353
A.2.3	Congruent, similar triangles	354
A.2.4	Triangle inequality, degenerate triangles	355
A.2.5	Quadrilaterals, n -gons	357
A.3	Similar triangles	359
A.4	Areas	362
A.4.1	Rectangles	362
A.4.2	Triangles	363

A.4.3	Circles	366
A.4.4	Cones	371
A.4.5	Spheres	373
A.5	Volumes	374
A.5.1	Blocks, etc.	374
A.5.2	Cones, pyramids	376
A.5.3	Spheres	379
A.6	The Pythagorean theorem	380
A.6.1	The theorem	380
A.6.2	General form of triples	382
A.6.3	Five proofs	384
A.6.4	The converse	386
A.7	Special triangles	387
A.8	Exercise solutions	391
B	Benefits of using letters	399
B.1	Distance to the horizon	399
B.2	Advantages of letters	406
B.3	Exercise solutions	411
	Glossary of notation	413

Volume 2

Preface		ix
7	General functions	425
7.1	The basics	426
7.2	Quadratic functions	428
7.3	General functions	433
7.4	Properties and terminology	438
7.5	Inverse	452
7.5.1	Definition	452
7.5.2	The f^{-1} notation	455
7.5.3	Existence of the inverse	459
7.5.4	Inverse of the inverse	460
7.5.5	Self inverse	462
7.6	Exercise solutions	465
8	Quadratic equations	476
8.1	Completing the square	476
8.2	Functions and plots	484
8.3	The quadratic formula	488
8.4	The discriminant	502

8.4.1	The three cases	502
8.4.2	Maxima and minima	505
8.5	The imaginary number i	510
8.5.1	Definition and properties	510
8.5.2	Examples	515
8.6	Exercise solutions	520
9	Logs and exponentials	543
9.1	Logs	543
9.2	Further considerations	548
9.2.1	Fractional and negative logs	548
9.2.2	Allowed values	550
9.2.3	Inverses	552
9.3	Log rules	554
9.4	Euler's number, e	561
9.5	Raising e to a power	564
9.6	Compound interest	568
9.7	Exercise solutions	575
10	Means, sequences, and series	583
10.1	Means	583
10.1.1	Arithmetic mean	583
10.1.2	Geometric mean	585
10.2	AM-GM inequality	588
10.2.1	Derivation	588
10.2.2	If and only if	591
10.2.3	Maxima and minima	594
10.3	Arithmetic sequences	597
10.4	Arithmetic series	600
10.5	Geometric sequences	605
10.6	Geometric series	610
10.6.1	Infinite geometric series	610
10.6.2	Finite geometric series	620
10.7	Exercise solutions	624
11	Miscellaneous topics	654
11.1	Proof by induction	654
11.2	Counting groups inductively	662
11.3	Deductive proof of sums	665
11.4	Areas	669
11.5	Volumes	674
11.6	A useful approximation	678
11.7	Exercise solutions	689

12 Slopes of general functions	707
12.1 Definition of slope	708
12.2 Procedure for finding the slope	711
12.2.1 The main idea	711
12.2.2 Application to parabola	713
12.3 Higher powers of x	717
12.3.1 Cubic	717
12.3.2 General x^n	721
12.4 Multiples and sums	723
12.4.1 Multiples	723
12.4.2 Sums	724
12.5 Standard calculus method	728
12.6 Exercise solutions	735
C Binomial coefficients	746
C.1 Unordered groups, rewritten	746
C.2 Binomial coefficients	749
C.3 The infinite series for e^x	756
C.3.1 The form of the series	756
C.3.2 Derivation of the series	757
C.4 Exercise solutions	764
Glossary of notation	770